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# Графический метод.

При ограничениях:1. Построим область допустимых решений, т.е. решим графически систему неравенств. Для этого построим каждую прямую и определим полуплоскости, заданные неравенствами (рис 1 и 2)

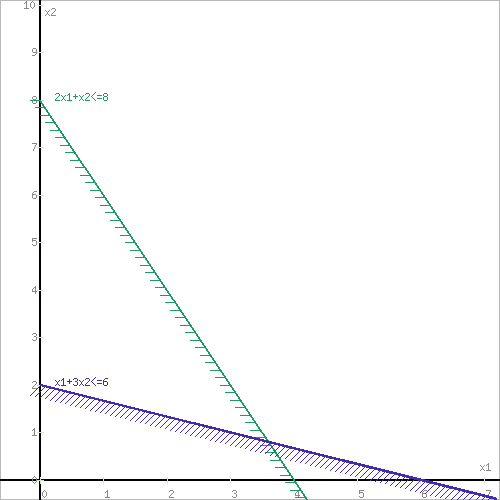


Рис 1. Ограничения, построенные по двум точкам

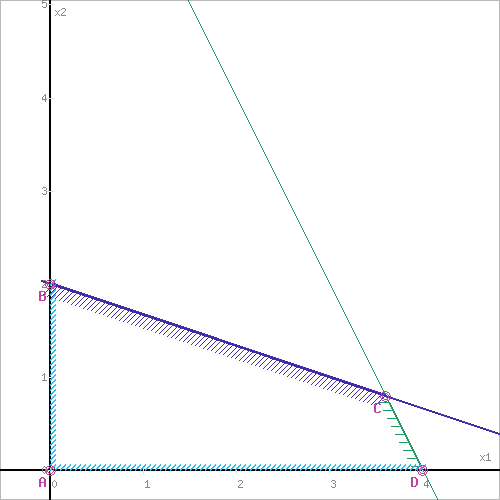


Рис 2. Границы ОДР

2. Рассмотрим целевую функцию F(x1,x2), найдем и построим ее градиент

Вектор-градиент, составленный из коэффициентов целевой функции, указывает направление максимизации F(X). Начало вектора – точка (0; 0), конец – точка (3;2). Построим прямую, нормальную к полученной и будем двигать ее вдоль вектора градиента. Так, точкой максимума будет считаться точка, в которой прямая покидает пределы области на рис 3б очевидно, что это точка C, а точкой минимума считается та точка, в которой прямая первый раз входит в пределы области, таким образом, из рисунка 3а очевидно, что такой точкой является точка начала координат (0,0)



Рис 3а. «Минимум функции»



Рис 3б «Максимум функции»

Для нахождения координат точки С обратим внимание, что она образована точкой пересечения ограничений 1 и 2, решим систему вида:

→ → →

Методом подстановки в любое из равенств получаем, что x1=3,6, следовательно координаты максимума функции F(x1,x2) = (3,6;0.8)

Путем подстановки полученных координат можем найти значение целевой функции в точках max и min (Табл. 1)

|  |  |  |  |
| --- | --- | --- | --- |
| Название | X1 | X2 | F(x1,,x2) |
| Fmin | 0 | 0 | 0 |
| Fmax | 3,6 | 0,8 | 12,4 |

Табл. 1 «Результаты»

# Симплекс-метод

А)

При ограничениях:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | X1 | X2 | X3 | X4 |
| X3 | 0 | 8 | ***2*** | 1 | 1 | 0 |
| X4 | 0 | 6 | 1 | 3 | 0 | 1 |
| Δ | F=0 |  | 3 | 2 | 0 | 0 |

Min{}→ 4

Текущий план (0,0,8,6)

Перестроим симплекс таблицу в новый базис

И пересчитаем коэффициенты по правилу прямоугольника

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | X1 | X2 | X3 | X4 |
| X1 | 3 | 4 | 1 | 0.5 | 0.5 | 0 |
| X4 | 0 | 2 | 0 | 2.5 | -0.5 | 1 |
| Δ | F=12 |  | 3 | 2 | 0 | 0 |

Пересчитаем дельты:

Д1=0 заведомо т к базис

Д4=0 заведомо т к базис

Д2=2-(0.5\*3+0\*2,5) = 2-1.5=0.5

Д3=0-(0.5\*3+0\*(-0.5)) = 0-1.5=-1.5

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | X1 | X2 | X3 | X4 |
| X1 | 3 | 4 | 1 | 0.5 | 0.5 | 0 |
| X4 | 0 | 2 | 0 | ***2.5*** | -0.5 | 1 |
| Δ | F=12 |  | 0 | 0.5 | -1.5 | 0 |

Min{}

Текущий план (4,0,0,2)

Перестроим симплекс таблицу в новый базис

И пересчитаем коэффициенты по правилу прямоугольника

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | X1 | X2 | X3 | X4 |
| X1 | 3 | 3.6 | 1 | 0 | 0.6 | -0.2 |
| X2 | 2 | 0.8 | 0 | 1 | -0.2 | 0.4 |
| Δ | F=12.4 |  | 0 | 0.5 | -1.5 | 0 |

Пересчитываем дельты:

Д3: 0-(3\*0.6+2\*(-0.2)) = 0-(1.8-0.4) = -1.4

Д4: 0-(3\*(-0.2)+2\*0.4) = 0-(-0.6+0.8) = 0-0.2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | X1 | X2 | X3 | X4 |
| X1 | 3 | 3.6 | 1 | 0 | 0.6 | -0.2 |
| X2 | 2 | 0.8 | 0 | 1 | -0.2 | 0.4 |
| Δ | F=12.4 |  | 0 | 0 | -1.4 | -0.2 |

План оптимален  
**X1 = 3.6; X2 =0.8**

**ОПТ решение: (3.6,0.8,0,0)**

**Fmax = 3\*3.6+2\*0.8=12.4**

# Метод искусственного базиса

Б) F = x1 + 3x2 → max

Приводим к каноническому виду:

|  |  |  |  |
| --- | --- | --- | --- |
| A1 | A2 | A3 | A4 |
| (3,2) | (4,-1) | (-1,0) | (0,1) |

Базис: А4, Аy1

Поставим задачу G  
G=-y1 →max

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 0 | 0 | 0 | 0 | -1 |
| Базис | C | B | A1 | A2 | A3 | A4 | Ay1 |
| Ay1 | -1 | 12 | 3 | **4** | -1 | 0 | 1 |
| A4 | 0 | 6 | 2 | -1 | 0 | 1 | 0 |
| Δ | G=-12 |  | 3 | 4 | -1 | 0 | 0 |

Текущий план: (0,0,0,6,12)

Д1:0-(-3)=3

Д2:0-(-4)=4

Д3:0-(1)=-1

Д4:0-0=0

Д5:-1-(-1)=0

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 0 | 0 | 0 | 0 | -1 |
| Базис | C | B | A1 | A2 | A3 | A4 | Ay1 |
| A2 | 0 | 3 | 3/4 | 1 | -1/4 | 0 | 1/4 |
| A4 | 0 | 9 | 11/4 | 0 | -1/4 | 1 | 1/4 |
| Δ | G=0 |  | 0 | 0 | 0 | 0 | -1 |

ОПТ план: (0,3,0,9,0)

Данный план является оптимальным, базис А2, А4 явл. базисом исходной задачи т к G=0

Решаем:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | A1 | A2 | A3 | A4 |
| A2 | 2 | 3 | 3/4 | 1 | -1/4 | 0 |
| A4 | 0 | 9 | 11/4 | 0 | -1/4 | 1 |
|  | F=6 |  | 1.5 | 0 | 0.5 | 0 |

Д1: 3-(2\*3/4)=3-1.5=1.5

Д2: 2-(2) = 0

Д3:0-(-0.5) = 0.5

Д4 = 0-(0)=0

**Критерий отсутствия решения для вектора А3 выполнен  
функция не ограничена сверху в ОДР**

В) F = x1 + 2x2 → max

|  |  |  |  |
| --- | --- | --- | --- |
| A1 | A2 | A3 | A4 |
| (1,-2) | (-6,2) | (-1,0) | (0,-1) |

G =-y1 -y2 →max

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 0 | 0 | 0 | 0 | -1 | -1 |
| Базис | C | B | А1 | А2 | А3 | А4 | Аy1 | Аy2 |
| Аy1 | -1 | 6 | 1 | -6 | -1 | 0 | 1 | 0 |
| Аy2 | -1 | 2 | -2 | 1 | 0 | -1 | 0 | 1 |
|  | G=-8 |  | -1 | -5 | -1 | -1 | 0 | 0 |

Д1:0-(-1+2)=-1

Д2: 0-(6-1)=-5

Д3:0-(1)=-1

Д4: 0-(1)=-1

Д5:-1-(-1) =0

Д6:-1-(-1) =0

Опт решение (0,0,0,0,6,2)

**ОПТ решение, но G<0 => ОДР пуста**

# Двойственные задачи ЛП

А)

При ограничениях:

Двойственная задача к данной:

G=8y1+6y2 → min

**ОПТ решение: (3.6,0.8,0,0)**

**Fmax = 3\*3.6+2\*0.8=12.4**

**По теореме 2**

**Y1=1.4; Y2=0.2**

**Gmin = 12.4**

**По теореме 3**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | X1 | X2 | X3 | X4 |
| X1 | 3 | 3.6 | 1 | 0 | 0.6 | -0.2 |
| X2 | 2 | 0.8 | 0 | 1 | -0.2 | 0.4 |
| Δ | F=12.4 |  | 0 | 0 | -1.4 | -0.2 |

Оптимальная симплекс таблица

Y\*=СB\*AB-1

(3,2)\*=(1,4;0.2)

**Y1=1.4; Y2=0.2**

**Gmin = 12.4**

# Экономическая интерпретация двойственной задачи

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Сырьё** | **Технологические коэффициенты** | | | | **Запасы** |
|  | **A** | **B** | **C** | **D** |  |
| металл | 5 | 1 | 0 | 2 | 1000 |
| пластмасса | 4 | 2 | 2 | 1 | 600 |
| резина | 1 | 0 | 2 | 1 | 150 |
| **Прибыль**(руб) | 6 | 2 | 3 | 4 |  |

количество продукции j вида,

, , , , , ,

А5, А6, А7 – базис

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 6 | 2 | 3 | 4 | 0 | 0 | 0 |  |
| **Базис** | **сбаз** | **b** | **А1** | **А2** | **А3** | **А4** | **А5** | **А6** | **А7** |  |
| А5 | 0 | 1000 | 5 | 1 | 0 | 2 | 1 | 0 | 0 | 200 |
| А6 | 0 | 600 | 4 | 2 | 2 | 1 | 0 | 1 | 0 | 150 |
| А7 | 0 | 150 | 1 | 0 | 2 | 1 | 0 | 0 | 1 | 150 |
|  | f(*x*оп) | 0 | 6 | 2 | 3 | 4 | 0 | 0 | 0 |  |

– не оптимальное решение, критерий отсутствия решения не выполняется

А5, А6, А1 – базис

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 6 | 2 | 3 | 4 | 0 | 0 | 0 |  |
| **Базис** | **сбаз** | **b** | **А1** | **А2** | **А3** | **А4** | **А5** | **А6** | **А7** |  |
| А5 | 0 | 250 | 0 | 1 | -10 | -3 | 1 | 0 | -5 | 250 |
| А6 | 0 | 0 | 0 | 2 | -6 | -3 | 0 | 1 | -4 | 0 |
| А1 | 6 | 150 | 1 | 0 | 2 | 1 | 0 | 0 | 1 |  |
|  | f(*x*оп) | 900 | 0 | 2 | -9 | -2 | 0 | 0 | -6 |  |

– не оптимальное решение

А5, А2, А1 – базис

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 6 | 2 | 3 | 4 | 0 | 0 | 0 |
| **Базис** | **сбаз** | **b** | **А1** | **А2** | **А3** | **А4** | **А5** | **А6** | **А7** |
| А5 | 0 | 250 | 0 | 0 | -7 | -1,5 | 1 | -0,5 | -3 |
| А2 | 2 | 0 | 0 | 1 | -3 | -1,5 | 0 | 0,5 | -2 |
| А1 | 6 | 150 | 1 | 0 | 2 | 1 | 0 | 0 | 1 |
|  | f(*x*оп) | 900 | 0 | 0 | -3 | 1 | 0 | -1 | -2 |

– не оптимальное решение

А5, А2, А4 – базис

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 6 | 2 | 3 | 4 | 0 | 0 | 0 |
| **Базис** | **сбаз** | **b** | **А1** | **А2** | **А3** | **А4** | **А5** | **А6** | **А7** |
| ёА5 | 0 | 475 | 1,5 | 0 | -4 | 0 | 1 | -0,5 | -1,5 |
| А2 | 2 | 225 | 1,5 | 1 | 0 | 0 | 0 | 0,5 | -0,5 |
| А4 | 4 | 150 | 1 | 0 | 2 | 1 | 0 | 0 | 1 |
|  | f(*x*оп) | 1050 | -1 | 0 | -5 | 0 | 0 | -1 | -3 |

– оптимальное решение

Поставим задачу двойственную к данной:

*Теорема 2:*

Выясним, какие равны 0 при подстановке оптимального решения в ограничения:

=> , так как 1 ограничение <0

=> , так как 2 ограничение =0

= 0 => , так как 3 ограничение = 0

Выясним, какие ограничения двойственной задачи в оптимальной точке выполняются как равенства:

⬄

*Теорема 3:*

**Выводы:**



= 0

Второе и третье ограничения выполнились как равенства => ресурсы 2 и 3 вида полностью использовались при оптимальном плане и являются дефицитными.

Первое ограничение выполнилось как строгое неравенство => ресурс 1 вида не является дефицитным. Его остаток

Второе и четвертое ограничения выполнились как равенства => двойственные оценки ресурсов, используемых для производства единицы продукции 2 и 4 вида в точности равны прибыли => целесообразно производить эти экономические изделия. (= 225 > 0, = 150 > 0)

Первое и третье ограничения – строгие неравенства => суммарные оценки сырья > получаемой прибыли =>производить эти виды продукции экономически нецелесообразно ()

1. Увеличение сырья 2 вида (пластмасса) на 1 единицу приведет к получению нового плана производства, при этом прибыль увеличивается на 1 и станет равна 1050+1=1051. Произойдет это за счет увеличения продукции В на 0,5, при этом остатки сырья 1 вида (металл) уменьшатся на 0,5.

Увеличение сырья 3 вида (резина) на 1 единицу приведет к получению нового плана производства, при этом прибыль увеличивается на 3 и станет равна 1050+3=1053. Произойдет это за счет уменьшения выпуска продукции B на 0,5 и увеличения выпуска изделий вида D на 1, при этом остатки сырья 1 вида уменьшатся на 1,5.

# Анализ устойчивости двойственных оценок

=

475 +

⬄

⬄

– входит в зону устойчивости

Первый вид ресурса в оптимальном плане недоиспользован, является недефицитным. Увеличение данного ресурса приведет лишь к росту его остатка.

При этом изменений в оптимальном плане не будет, так как

Как изменится план выпуска продукции, если ?

Оптимальный план не изменится, причем прибыль станет равной 1050+100+450 = 1600

# Транспортная задача

Имеем транспортную задачу заданную функцией:

Составим матрицу:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | B1 | B2 | B3 | B4 | B5 | B6 | Запасы |
| A1 | 1 | 1 | 3 | 4 | 2 | 7 | 30 |
| A2 | 3 | 2 | 1 | 5 | 4 | 5 | 35 |
| A3 | 1 | 4 | 6 | 3 | 5 | 2 | 40 |
| A4 | 5 | 7 | 4 | 2 | 4 | 3 | 45 |
| Потребности | 15 | 30 | 20 | 35 | 25 | 25 | 150/150 |

## Метод северо-западного угла

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | B1 | B2 | B3 | B4 | B5 | B6 | Запасы |
| A1 | ~~1~~**15** | ~~1~~**15** | ~~3~~ | ~~4~~ | ~~2~~ | ~~7~~ | ~~30/15~~/0 |
| A2 | ~~3~~ | ~~2~~**15** | ~~1~~**20** | ~~5~~ | ~~4~~ | ~~5~~ | 35/~~20~~/0 |
| A3 | ~~1~~ | ~~4~~ | ~~6~~ | ~~3~~**35** | ~~5~~**5** | ~~2~~ | ~~40~~/~~5~~/0 |
| A4 | ~~5~~ | ~~7~~ | ~~4~~ | ~~2~~ | ~~4~~**20** | 3**25** | ~~45/25~~/0 |
| Потребности | ~~15/~~0 | ~~30~~/~~15~~/0 | ~~20~~/0 | ~~35~~/0 | ~~25~~/~~20~~/0 | ~~25~~/0 | 150/150 |

F=15+15+30+20+105+25+80+75=365

## Метод минимальных коэффициентов

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | B1 | B2 | B3 | B4 | B5 | B6 | Запасы |
| A1 | 1**15** | 1**15** | 3 | 4 | 2 | 7 | 30/15/0 |
| A2 | 3 | 2**15** | 1**20** | 5 | 4 | 5 | 35/15/0 |
| A3 | 1 | 4 | 6 | 3 | 5**15** | 2**25** | 40/15/0 |
| A4 | 5 | 7 | 4 | 2**35** | 4**10** | 3 | 45/10/0 |
| Потребности | 15/0 | 30/15/0 | 20/0 | 35/0 | 25 | 25/0 | 150/150 |

F(x) = 1\*15 + 1\*15 + 2\*15 + 1\*20 + 5\*15 + 2\*25 + 2\*35 + 4\*10 = 315

План по второму методу получился лучше, поэтому возьмем его в качестве начального.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | B1 | B2 | B3 | B4 | B5 | B6 | Ui |
| A1 | 1**15** | 1**15** | 3 | 4 | 2 | 7 | U1 |
| A2 | 3 | 2**15** | 1**20** | 5 | 4 | 5 | U2 |
| A3 | 1**0** | 4 | 6 | 3 | 5**15** | 2**25** | U3 |
| A4 | 5 | 7 | 4 | 2**35** | 4**10** | 3 | U4 |
| Vj | V1 | V2 | V3 | V4 | V5 | V6 |  |

Пусть =>

Опорный план не является оптимальным, так как существуют оценки свободных клеток, для которых ui + vj > cij

(1;5): 0 + 5 > 2; ∆15 = 0 + 5 - 2 = 3 > 0

(2;5): 1 + 5 > 4; ∆25 = 1 + 5 - 4 = 2 > 0

max(3,2) = 3

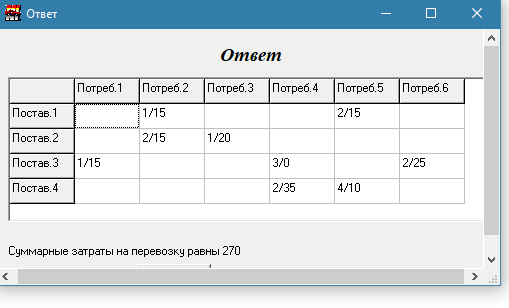
|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | B1 | B2 | B3 | B4 | B5 | B6 | Ui |
| A1 | 1**15(-)** | 1**15** | 3 | 4 | 2**+** | 7 | U1 |
| A2 | 3 | 2**15** | 1**20** | 5 | 4 | 5 | U2 |
| A3 | 1**0(+)** | 4 | 6 | 3 | 5**15(-)** | 2**25** | U3 |
| A4 | 5 | 7 | 4 | 2**35** | 4**10** | 3 | U4 |
| Vj | V1 | V2 | V3 | V4 | V5 | V6 |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | B1 | B2 | B3 | B4 | B5 | B6 | Ui |
| A1 | 1 | 1**15** | 3 | 4 | 2**15** | 7 | U1 |
| A2 | 3 | 2**15** | 1**20** | 5 | 4 | 5 | U2 |
| A3 | 1**15** | 4 | 6 | 3 | 5**0** | 2**25** | U3 |
| A4 | 5 | 7 | 4 | 2**35** | 4**10** | 3 | U4 |
| Vj | V1 | V2 | V3 | V4 | V5 | V6 |  |

Пусть =>

Опорный план является оптимальным, так все оценки свободных клеток удовлетворяют условию ui + vj ≤ cij.

Минимальные затраты составят: F(x) = 1\*15 + 2\*15 + 2\*15 + 1\*20 + 1\*15 + 2\*25 + 2\*35 + 4\*10 = 270



# Метод отсечения Гомори

При ограничениях:



Рис 3а. «Минимум функции»



Рис 3б «Максимум функции»

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | X1 | X2 | X3 | X4 |
| X3 | 0 | 8 | ***2*** | 1 | 1 | 0 |
| X4 | 0 | 6 | 1 | 3 | 0 | 1 |
| Δ | F=0 |  | 3 | 2 | 0 | 0 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | X1 | X2 | X3 | X4 |
| X1 | 3 | 4 | 1 | 0.5 | 0.5 | 0 |
| X4 | 0 | 2 | 0 | ***2.5*** | -0.5 | 1 |
| Δ | F=12 |  | 0 | 0.5 | -1.5 | 0 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 |
| Базис | C | B | X1 | X2 | X3 | X4 |
| X1 | 3 | 3.6 | 1 | 0 | 0.6 | -0.2 |
| X2 | 2 | 0.8 | 0 | 1 | -0.2 | 0.4 |
| Δ | F=12.4 |  | 0 | 0 | -1.4 | -0.2 |

План оптимален  
**X1 = 3.6; X2 =0.8**

**ОПТ решение: (3.6,0.8,0,0)**

**Fmax = 3\*3.6+2\*0.8=12.4**

не удовлетворяет требованию целочисленности => строим правильное отсечение (отсекаем по , так как у нее наибольшая дробная часть)

Выразим x4;x3 через каноническую систему:

Подставим в полученное доп ограничение:

Добавим новое ограничение к ранее имевшимся

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 | 0 |
| Базис | C | B | A1 | A2 | A3 | A4 | A5 |
| A3 | 0 | 8 | **2** | 1 | 1 | 0 | 0 |
| A4 | 0 | 6 | 1 | 3 | 0 | 1 | 0 |
| A5 | 0 | 4 | 1 | 1 | 0 | 0 | 1 |
| Δ | F=0 |  | 3 | 2 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 | 0 |
| Базис | C | B | A1 | A2 | A3 | A4 | A5 |
| A1 | 3 | 4 | 1 | 1/2 | 1/2 | 0 | 0 |
| A4 | 0 | 2 | 0 | 5/2 | -1/2 | 1 | 0 |
| A5 | 0 | 0 | 0 | **1/2** | -1/2 | 0 | 1 |
| Δ | F=12 |  | 0 | 1/2 | -3/2 | 0 | 0 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 3 | 2 | 0 | 0 | 0 |
| Базис | C | B | A1 | A2 | A3 | A4 | A5 |
| A1 | 3 | 4 | 1 | 0 | 1 | 0 | -1 |
| A4 | 0 | 2 | 0 | 0 | 2 | 1 | -5 |
| A2 | 2 | 0 | 0 | 1 | -1 | 0 | 2 |
| Δ | F=12 |  | 0 | 0 | -1 | 0 | -1 |

x1 = 4, x2 = 0

F(X) = 3\*4 + 2\*0 = 12

# Задача Коммивояжёра

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 13 | 4 | 14 | 2 | 3 | 1 | 1 | 14 | 12 |
| 2 | 13 | ∞ | 3 | 10 | 1 | 15 | 11 | 8 | 4 | 9 |
| 3 | 4 | 3 | ∞ | 3 | 13 | 8 | 15 | 13 | 5 | 3 |
| 4 | 14 | 10 | 3 | ∞ | 10 | 8 | 6 | 15 | 7 | 7 |
| 5 | 2 | 1 | 13 | 10 | ∞ | 6 | 6 | 9 | 11 | 9 |
| 6 | 3 | 15 | 8 | 8 | 6 | ∞ | 9 | 2 | 9 | 5 |
| 7 | 1 | 11 | 15 | 6 | 6 | 9 | ∞ | 1 | 10 | 6 |
| 8 | 1 | 8 | 13 | 15 | 9 | 2 | 1 | ∞ | 9 | 11 |
| 9 | 14 | 4 | 5 | 7 | 11 | 9 | 10 | 9 | ∞ | 3 |
| 10 | 12 | 9 | 3 | 7 | 9 | 5 | 6 | 11 | 3 | ∞ |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | min |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 13 | 4 | 14 | 2 | 3 | 1 | 1 | 14 | 12 | 1 | 1 | ∞ | 12 | 3 | 13 | 1 | 2 | 0 | 0 | 13 | 11 |
| 2 | 13 | ∞ | 3 | 10 | 1 | 15 | 11 | 8 | 4 | 9 | 1 | 2 | 12 | ∞ | 2 | 9 | 0 | 14 | 10 | 7 | 3 | 8 |
| 3 | 4 | 3 | ∞ | 3 | 13 | 8 | 15 | 13 | 5 | 3 | 3 | 3 | 1 | 0 | ∞ | 0 | 10 | 5 | 12 | 10 | 2 | 0 |
| 4 | 14 | 10 | 3 | ∞ | 10 | 8 | 6 | 15 | 7 | 7 | 3 | 4 | 11 | 7 | 0 | ∞ | 7 | 5 | 3 | 12 | 4 | 4 |
| 5 | 2 | 1 | 13 | 10 | ∞ | 6 | 6 | 9 | 11 | 9 | 1 | 5 | 1 | 0 | 12 | 9 | ∞ | 5 | 5 | 8 | 10 | 8 |
| 6 | 3 | 15 | 8 | 8 | 6 | ∞ | 9 | 2 | 9 | 5 | 2 | 6 | 1 | 13 | 6 | 6 | 4 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 1 | 11 | 15 | 6 | 6 | 9 | ∞ | 1 | 10 | 6 | 1 | 7 | 0 | 10 | 14 | 5 | 5 | 8 | ∞ | 0 | 9 | 5 |
| 8 | 1 | 8 | 13 | 15 | 9 | 2 | 1 | ∞ | 9 | 11 | 1 | 8 | 0 | 7 | 12 | 14 | 8 | 1 | 0 | ∞ | 8 | 10 |
| 9 | 14 | 4 | 5 | 7 | 11 | 9 | 10 | 9 | ∞ | 3 | 3 | 9 | 11 | 1 | 2 | 4 | 8 | 6 | 7 | 6 | ∞ | 0 |
| 10 | 12 | 9 | 3 | 7 | 9 | 5 | 6 | 11 | 3 | ∞ | 3 | 10 | 9 | 6 | 0 | 4 | 6 | 2 | 3 | 8 | 0 | ∞ |
|  |  |  |  |  |  |  |  |  |  |  |  | min | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 12 | 3 | 13 | 1 | 1 | 0 | 0 | 13 | 11 |
| 2 | 12 | ∞ | 2 | 9 | 0 | 13 | 10 | 7 | 3 | 8 |
| 3 | 1 | 0 | ∞ | 0 | 10 | 4 | 12 | 10 | 2 | 0 |
| 4 | 11 | 7 | 0 | ∞ | 7 | 4 | 3 | 12 | 4 | 4 |
| 5 | 1 | 0 | 12 | 9 | ∞ | 4 | 5 | 8 | 10 | 8 |
| 6 | 1 | 13 | 6 | 6 | 4 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 0 | 10 | 14 | 5 | 5 | 7 | ∞ | 0 | 9 | 5 |
| 8 | 0 | 7 | 12 | 14 | 8 | 0 | 0 | ∞ | 8 | 10 |
| 9 | 11 | 1 | 2 | 4 | 8 | 5 | 7 | 6 | ∞ | 0 |
| 10 | 9 | 6 | 0 | 4 | 6 | 1 | 3 | 8 | 0 | ∞ |

Сумма производящих констант: 1+1+3+3+1+2+1+1+3+3+1 = 20

Оценка множества: w(G0) = 20

d(1,7) = 0 + 0 = 0;

d(1,8) = 0 + 0 = 0;

d(2,5) = 2 + 1 = 3;

d(3,2) = 0 + 0 = 0;

d(3,4) = 0 + 4 = 4;

d(3,10) = 0 + 0 = 0;

d(4,3) = 3 + 0 = 3;

d(5,2) = 1 + 0 = 1;

d(6,8) = 1 + 0 = 1;

d(7,1) = 0 + 0 = 0;

d(7,8) = 0 + 0 = 0;

d(8,1) = 0 + 0 = 0;

d(8,6) = 0 + 1 = 1;

d(8,7) = 0 + 0 = 0;

d(9,10) = 1 + 0 = 1;

d(10,3) = 0 + 0 = 0;

d(10,9) = 0 + 2 = 2;

Qmax = Q(3,4) = 4

Пара для ветвления:

Нижняя граница множества: w({3,4})=

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 12 | 3 | 1 | 1 | 0 | 0 | 13 | 11 |
| 2 | 12 | ∞ | 2 | 0 | 13 | 10 | 7 | 3 | 8 |
| 4 | 11 | 7 | ∞ | 7 | 4 | 3 | 12 | 4 | 4 |
| 5 | 1 | 0 | 12 | ∞ | 4 | 5 | 8 | 10 | 8 |
| 6 | 1 | 13 | 6 | 4 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 0 | 10 | 14 | 5 | 7 | ∞ | 0 | 9 | 5 |
| 8 | 0 | 7 | 12 | 8 | 0 | 0 | ∞ | 8 | 10 |
| 9 | 11 | 1 | 2 | 8 | 5 | 7 | 6 | ∞ | 0 |
| 10 | 9 | 6 | 0 | 6 | 1 | 3 | 8 | 0 | ∞ |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 | min |  | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 12 | 3 | 1 | 1 | 0 | 0 | 13 | 11 | 0 | 1 | ∞ | 12 | 3 | 1 | 1 | 0 | 0 | 13 | 11 |
| 2 | 12 | ∞ | 2 | 0 | 13 | 10 | 7 | 3 | 8 | 0 | 2 | 12 | ∞ | 2 | 0 | 13 | 10 | 7 | 3 | 8 |
| 4 | 11 | 7 | ∞ | 7 | 4 | 3 | 12 | 4 | 4 | 3 | 4 | 8 | 4 | ∞ | 4 | 1 | 0 | 9 | 1 | 1 |
| 5 | 1 | 0 | 12 | ∞ | 4 | 5 | 8 | 10 | 8 | 0 | 5 | 1 | 0 | 12 | ∞ | 4 | 5 | 8 | 10 | 8 |
| 6 | 1 | 13 | 6 | 4 | ∞ | 7 | 0 | 7 | 3 | 0 | 6 | 1 | 13 | 6 | 4 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 0 | 10 | 14 | 5 | 7 | ∞ | 0 | 9 | 5 | 0 | 7 | 0 | 10 | 14 | 5 | 7 | ∞ | 0 | 9 | 5 |
| 8 | 0 | 7 | 12 | 8 | 0 | 0 | ∞ | 8 | 10 | 0 | 8 | 0 | 7 | 12 | 8 | 0 | 0 | ∞ | 8 | 10 |
| 9 | 11 | 1 | 2 | 8 | 5 | 7 | 6 | ∞ | 0 | 0 | 9 | 11 | 1 | 2 | 8 | 5 | 7 | 6 | ∞ | 0 |
| 10 | 9 | 6 | 0 | 6 | 1 | 3 | 8 | 0 | ∞ | 0 | 10 | 9 | 6 | 0 | 6 | 1 | 3 | 8 | 0 | ∞ |
|  |  |  |  |  |  |  |  |  |  |  | MIN | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Сумма производящих констант: 3

Оценка множества w({3,4})=

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| min |  | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0 | 1 | ∞ | 12 | 3 | 1 | 1 | 0 | 0 | 13 | 11 |
| 2 | 2 | 12 | ∞ | 2 | 0 | 13 | 10 | 7 | 3 | 8 |
| 1 | 4 | 8 | 4 | ∞ | 4 | 1 | 0 | 9 | 1 | 1 |
| 1 | 5 | 1 | 0 | 12 | ∞ | 4 | 5 | 8 | 10 | 8 |
| 1 | 6 | 1 | 13 | 6 | 4 | ∞ | 7 | 0 | 7 | 3 |
| 0 | 7 | 0 | 10 | 14 | 5 | 7 | ∞ | 0 | 9 | 5 |
| 0 | 8 | 0 | 7 | 12 | 8 | 0 | 0 | ∞ | 8 | 10 |
| 1 | 9 | 11 | 1 | 2 | 8 | 5 | 7 | 6 | ∞ | 0 |
| 0 | 10 | 9 | 6 | 0 | 6 | 1 | 3 | 8 | 0 | ∞ |
|  | MIN | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 1 | 1 |

Qmax = Q(2,5) = 3

Нижняя граница множества: w({2,5})=

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1 | 2 | 3 | 6 | 7 | 8 | 9 | 10 | min |  | 1 | 2 | 3 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 12 | 3 | 1 | 0 | 0 | 13 | 11 | 0 | 1 | ∞ | 12 | 3 | 1 | 0 | 0 | 13 | 11 |
| 4 | 8 | 4 | ∞ | 1 | 0 | 9 | 1 | 1 | 0 | 4 | 8 | 4 | ∞ | 1 | 0 | 9 | 1 | 1 |
| 5 | 1 | ∞ | 12 | 4 | 5 | 8 | 10 | 8 | 1 | 5 | 0 | ∞ | 11 | 3 | 4 | 7 | 9 | 7 |
| 6 | 1 | 13 | 6 | ∞ | 7 | 0 | 7 | 3 | 0 | 6 | 1 | 13 | 6 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 0 | 10 | 14 | 7 | ∞ | 0 | 9 | 5 | 0 | 7 | 0 | 10 | 14 | 7 | ∞ | 0 | 9 | 5 |
| 8 | 0 | 7 | 12 | 0 | 0 | ∞ | 8 | 10 | 0 | 8 | 0 | 7 | 12 | 0 | 0 | ∞ | 8 | 10 |
| 9 | 11 | 1 | 2 | 5 | 7 | 6 | ∞ | 0 | 0 | 9 | 11 | 1 | 2 | 5 | 7 | 6 | ∞ | 0 |
| 10 | 9 | 6 | 0 | 1 | 3 | 8 | 0 | ∞ | 0 | 10 | 9 | 6 | 0 | 1 | 3 | 8 | 0 | ∞ |
|  |  |  |  |  |  |  |  |  | min |  | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

w({2,5})=

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 6 | 7 | 8 | 9 | 10 | min |
| 1 | ∞ | 11 | 3 | 1 | 0 | 0 | 13 | 11 | 0 |
| 4 | 8 | 3 | ∞ | 1 | 0 | 9 | 1 | 1 | 1 |
| 5 | 0 | ∞ | 11 | 3 | 4 | 7 | 9 | 7 | 3 |
| 6 | 1 | 12 | 6 | ∞ | 7 | 0 | 7 | 3 | 1 |
| 7 | 0 | 9 | 14 | 7 | ∞ | 0 | 9 | 5 | 0 |
| 8 | 0 | 6 | 12 | 0 | 0 | ∞ | 8 | 10 | 0 |
| 9 | 11 | 0 | 2 | 5 | 7 | 6 | ∞ | 0 | 0 |
| 10 | 9 | 5 | 0 | 1 | 3 | 8 | 0 | ∞ | 0 |
| min | 0 | 3 | 2 | 1 | 0 | 0 | 1 | 1 |  |

d(5,1) = 3 + 0 = 3;  
 d(9,2) = 0 + 3 = 3;

max: d(9,2)=3.

Нижняя граница множества: w({9,2})=

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 3 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 3 | 1 | 0 | 0 | 13 | 11 |
| 4 | 8 | ∞ | 1 | 0 | 9 | 1 | 1 |
| 5 | 0 | 11 | 3 | 4 | 7 | ∞ | 7 |
| 6 | 1 | 6 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 0 | 14 | 7 | ∞ | 0 | 9 | 5 |
| 8 | 0 | 12 | 0 | 0 | ∞ | 8 | 10 |
| 10 | 9 | 0 | 1 | 3 | 8 | 0 | ∞ |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 3 | 6 | 7 | 8 | 9 | 10 | min |  | 1 | 3 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 3 | 1 | 0 | 0 | 13 | 11 | 0 | 1 | ∞ | 3 | 1 | 0 | 0 | 13 | 11 |
| 4 | 8 | ∞ | 1 | 0 | 9 | 1 | 1 | 0 | 4 | 8 | ∞ | 1 | 0 | 9 | 1 | 1 |
| 5 | 0 | 11 | 3 | 4 | 7 | ∞ | 7 | 0 | 5 | 0 | 11 | 3 | 4 | 7 | ∞ | 7 |
| 6 | 1 | 6 | ∞ | 7 | 0 | 7 | 3 | 0 | 6 | 1 | 6 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 0 | 14 | 7 | ∞ | 0 | 9 | 5 | 0 | 7 | 0 | 14 | 7 | ∞ | 0 | 9 | 5 |
| 8 | 0 | 12 | 0 | 0 | ∞ | 8 | 10 | 0 | 8 | 0 | 12 | 0 | 0 | ∞ | 8 | 10 |
| 10 | 9 | 0 | 1 | 3 | 8 | 0 | ∞ | 0 | 10 | 9 | 0 | 1 | 3 | 8 | 0 | ∞ |
|  |  |  |  |  |  |  |  |  | min | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

h = 1

H(9,2) = 25 + 1 = 26 ≤ 28

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 3 | 6 | 7 | 8 | 9 | 10 | min |
| 1 | ∞ | 3 | 1 | 0 | 0 | 13 | 10 | 0 |
| 4 | 8 | ∞ | 1 | 0 | 9 | 1 | 0 | 0 |
| 5 | 0 | 11 | 3 | 4 | 7 | ∞ | 6 | 3 |
| 6 | 1 | 6 | ∞ | 7 | 0 | 7 | 2 | 1 |
| 7 | 0 | 14 | 7 | ∞ | 0 | 9 | 4 | 0 |
| 8 | 0 | 12 | 0 | 0 | ∞ | 8 | 9 | 0 |
| 10 | 9 | 0 | 1 | 3 | 8 | 0 | ∞ | 0 |
| min | 0 | 3 | 1 | 0 | 0 | 1 | 2 |  |

d(5,1) = 3 + 0 = 3;  
d(10,3) = 0 + 3 = 3;

max: d(10,3)=3.

H(10,3) = 26 + 3 =29

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 1 | 0 | 0 | 13 | 10 |
| 4 | 8 | 1 | 0 | 9 | 1 | 0 |
| 5 | 0 | 3 | 4 | 7 | ∞ | 6 |
| 6 | 1 | ∞ | 7 | 0 | 7 | 2 |
| 7 | 0 | 7 | ∞ | 0 | 9 | 4 |
| 8 | 0 | 0 | 0 | ∞ | 8 | 9 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 6 | 7 | 8 | 9 | 10 | min |  | 1 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 1 | 0 | 0 | 13 | 10 | 0 | 1 | ∞ | 1 | 0 | 0 | 13 | 10 |
| 4 | 8 | 1 | 0 | 9 | 1 | 0 | 0 | 4 | 8 | 1 | 0 | 9 | 1 | 0 |
| 5 | 0 | 3 | 4 | 7 | ∞ | 6 | 0 | 5 | 0 | 3 | 4 | 7 | ∞ | 6 |
| 6 | 1 | ∞ | 7 | 0 | 7 | 2 | 0 | 6 | 1 | ∞ | 7 | 0 | 7 | 2 |
| 7 | 0 | 7 | ∞ | 0 | 9 | 4 | 0 | 7 | 0 | 7 | ∞ | 0 | 9 | 4 |
| 8 | 0 | 0 | 0 | ∞ | 8 | 9 | 0 | 8 | 0 | 0 | 0 | ∞ | 8 | 9 |
|  |  |  |  |  |  |  |  | min | 0 | 0 | 0 | 0 | 1 | 0 |

H(10,3) = 26 + 1 = 27 ≤ 29

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 6 | 7 | 8 | 9 | 10 | min |
| 1 | ∞ | 1 | 0 | 0 | 12 | 10 | 0 |
| 4 | 8 | 1 | 0 | 9 | 0 | ∞ | 0 |
| 5 | 0 | 3 | 4 | 7 | ∞ | 6 | 3 |
| 6 | 1 | ∞ | 7 | 0 | 6 | 2 | 1 |
| 7 | 0 | 7 | ∞ | 0 | 8 | 4 | 0 |
| 8 | 0 | 0 | 0 | ∞ | 7 | 9 | 0 |
| min | 0 | 1 | 0 | 0 | 6 | 0 | 0 |

d(4,9) = 0 + 6 = 6;

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 6 | 7 | 8 | 9 | 10 | min |  | 1 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 1 | 0 | 0 | 12 | 10 | 0 | 1 | ∞ | 1 | 0 | 0 | 12 | 10 |
| 4 | 8 | 1 | 0 | 9 | ∞ | ∞ | 0 | 4 | 8 | 1 | 0 | 9 | ∞ | ∞ |
| 5 | 0 | 3 | 4 | 7 | ∞ | 6 | 0 | 5 | 0 | 3 | 4 | 7 | ∞ | 6 |
| 6 | 1 | ∞ | 7 | 0 | 6 | 2 | 0 | 6 | 1 | ∞ | 7 | 0 | 6 | 2 |
| 7 | 0 | 7 | ∞ | 0 | 8 | 4 | 0 | 7 | 0 | 7 | ∞ | 0 | 8 | 4 |
| 8 | 0 | 0 | 0 | ∞ | 7 | 9 | 0 | 8 | 0 | 0 | 0 | ∞ | 7 | 9 |
|  |  |  |  |  |  |  |  | min | 0 | 0 | 0 | 0 | 6 | 2 |

H(4,9) = 27+8 =35

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 6 | 7 | 8 | 10 | min |
| 1 | ∞ | 1 | 0 | 0 | 8 | 0 |
| 5 | 0 | 3 | 4 | 7 | 6 | 0 |
| 6 | 1 | ∞ | 7 | 0 | 0 | 0 |
| 7 | 0 | 7 | ∞ | 0 | 2 | 0 |
| 8 | 0 | 0 | 0 | ∞ | 7 | 0 |
| min | 0 | 0 | 0 | 2 | 2 |  |

H(4,9) = 27 + 2 = 29 ≤ 35

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 6 | 7 | 8 | 10 | min |
| 1 | ∞ | 1 | 0 | 0 | 8 | 0 |
| 5 | 0 | 3 | 4 | 7 | ∞ | 3 |
| 6 | 1 | ∞ | 7 | 0 | 0 | 0 |
| 7 | 0 | 7 | ∞ | 0 | 2 | 0 |
| 8 | 0 | 0 | 0 | ∞ | 7 | 0 |
| min | 0 | 1 | 0 | 0 | 2 |  |

d(5,1) = 3 + 0 = 3;

max: d(5,1)=3.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 6 | 7 | 8 | 10 | min |  | 1 | 6 | 7 | 8 | 10 |
| 1 | ∞ | 1 | 0 | 0 | 8 | 0 | 1 | ∞ | 1 | 0 | 0 | 8 |
| 5 | ∞ | 3 | 4 | 7 | ∞ | 3 | 5 | 0 | 3 | 4 | 7 | ∞ |
| 6 | 1 | ∞ | 7 | 0 | 0 | 0 | 6 | 1 | ∞ | 7 | 0 | 0 |
| 7 | 0 | 7 | ∞ | 0 | 2 | 0 | 7 | 0 | 7 | ∞ | 0 | 2 |
| 8 | 0 | 0 | 0 | ∞ | 7 | 0 | 8 | 0 | 0 | 0 | ∞ | 7 |
|  |  |  |  |  |  |  | min | 0 | 0 | 0 | 0 | 0 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 6 | 7 | 8 | 10 | min |
| 1 | 1 | 0 | 0 | 8 | 0 |
| 6 | ∞ | 7 | 0 | 0 | 0 |
| 7 | 7 | ∞ | 0 | 2 | 0 |
| 8 | 0 | 0 | ∞ | 7 | 0 |
| min | 0 | 0 | 0 | 0 | 0 |
|  |  |  |  |  |  |

H(5,1) = 29 + 0 = 29 ≤ 32

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 6 | 7 | 8 | 10 | min |
| 1 | 1 | 0 | 0 | 8 | 0 |
| 6 | ∞ | 7 | 0 | 0 | 0 |
| 7 | 7 | ∞ | 0 | 2 | 2 |
| 8 | 0 | 0 | ∞ | 7 | 0 |
| min | 1 | 0 | 0 | 2 | 0 |

d(1,7) = 0 + 0 = 0;   
d(1,8) = 0 + 0 = 0;  
 d(6,8) = 0 + 0 = 0;   
d(6,10) = 0 + 2 = 2;  
 d(7,8) = 2 + 0 = 2;  
 d(8,6) = 0 + 1 = 1;  
 d(8,7) = 0 + 0 = 0;

max: d(6,10)=2.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 6 | 7 | 8 | 10 | min |  | 6 | 7 | 8 | 10 |
| 1 | 1 | 0 | 0 | ∞ | 0 | 1 | 1 | 0 | 0 | ∞ |
| 6 | ∞ | 7 | 0 | ∞ | 0 | 6 | ∞ | 7 | 0 | ∞ |
| 7 | 7 | ∞ | 0 | 2 | 0 | 7 | 7 | ∞ | 0 | 2 |
| 8 | 0 | 0 | ∞ | 7 | 0 | 8 | 0 | 0 | ∞ | 7 |
|  |  |  |  |  |  | min | 0 | 0 | 0 | 2 |

H(6\*,10\*) = 29 + 2 = 31

|  |  |  |  |
| --- | --- | --- | --- |
|  | 6 | 7 | 8 |
| 1 | 1 | 0 | 0 |
| 7 | 7 | ∞ | 0 |
| 8 | 0 | 0 | ∞ |

H(6,10) = 29 + 0 = 29 ≤ 31

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 6 | 7 | 8 | min |
| 1 | ∞ | 0 | 0 | 0 |
| 7 | 7 | ∞ | 0 | 7 |
| 8 | 0 | 0 | ∞ | 0 |
| min | 7 | 0 | 0 |  |

d(7,8) = 7 + 0 = 7;

d(8,6) = 0 + 7 = 7;

max: d(7,8)=7.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 6 | 7 | 8 | min |  | 6 | 7 | 8 |
| 1 | ∞ | 0 | 0 | 0 | 1 | ∞ | 0 | 0 |
| 7 | 7 | ∞ | ∞ | 7 | 7 | 7 | ∞ | ∞ |
| 8 | 0 | 0 | ∞ | 0 | 8 | 0 | 0 | ∞ |
|  |  |  |  |  | min | 0 | 0 | 0 |

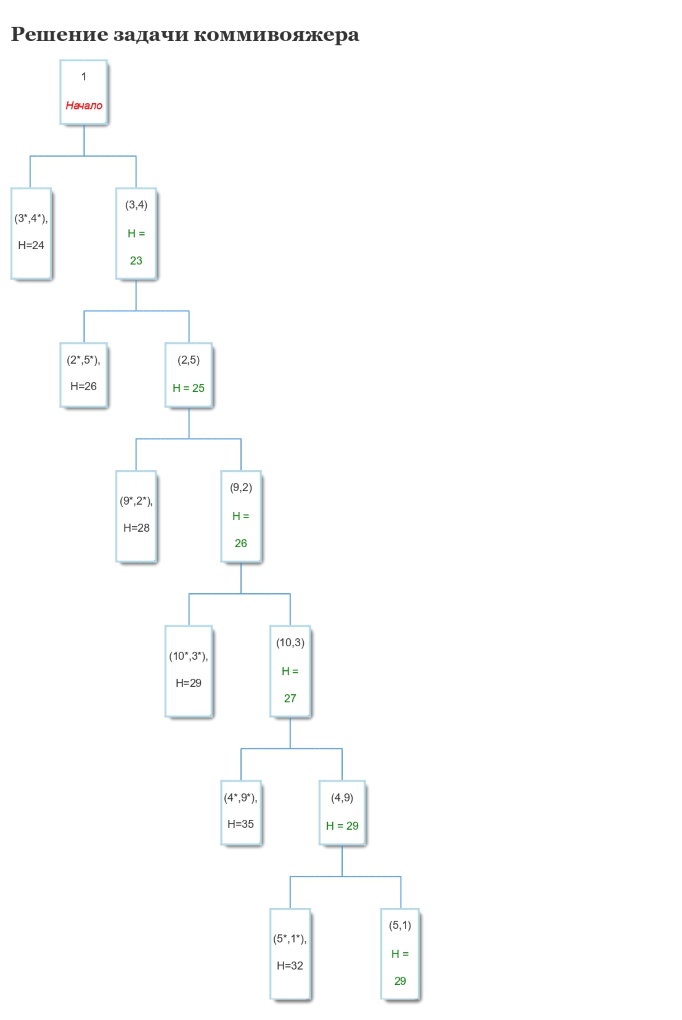
H(7\*,8\*) = 29 + 7 = 36

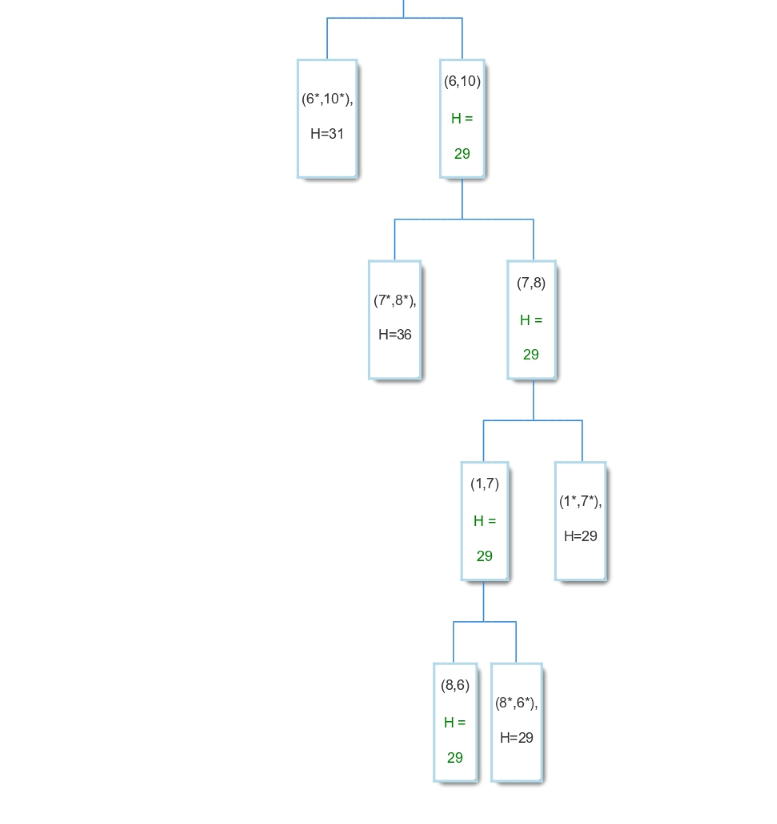
|  |  |  |  |
| --- | --- | --- | --- |
|  | 6 | 7 | min |
| 1 | ∞ | 0 | 0 |
| 7 | 7 | ∞ | 7 |
| 8 | 0 | 0 | 0 |
| min | 7 | 0 |  |
|  |  |  |  |

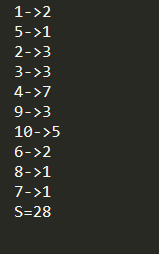
H(7,8) = 29 + 0 = 29 ≤ 36

Длина маршрута равна F(Mk) = 29

(3,4), (4,9), (9,2), (2,5), (5,1), (1,7), (7,8), (8,6), (6,10), (10,3),







Найденный план не оптимален, вернемся в точку, в которой первый раз оценка оказалась меньше итоговой

Исключим точку (3,4)=беск

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 12 | 3 | 13 | 1 | 1 | 0 | 0 | 13 | 11 |
| 2 | 12 | ∞ | 2 | 9 | 0 | 13 | 10 | 7 | 3 | 8 |
| 3 | 1 | 0 | ∞ | 0 | 10 | 4 | 12 | 10 | 2 | 0 |
| 4 | 11 | 7 | 0 | ∞ | 7 | 4 | 3 | 12 | 4 | 4 |
| 5 | 1 | 0 | 12 | 9 | ∞ | 4 | 5 | 8 | 10 | 8 |
| 6 | 1 | 13 | 6 | 6 | 4 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 0 | 10 | 14 | 5 | 5 | 7 | ∞ | 0 | 9 | 5 |
| 8 | 0 | 7 | 12 | 14 | 8 | 0 | 0 | ∞ | 8 | 10 |
| 9 | 11 | 1 | 2 | 4 | 8 | 5 | 7 | 6 | ∞ | 0 |
| 10 | 9 | 6 | 0 | 4 | 6 | 1 | 3 | 8 | 0 | ∞ |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | min |
| 1 | ∞ | 12 | 3 | 13 | 1 | 1 | 0 | 0 | 13 | 11 | 0 |
| 2 | 12 | ∞ | 2 | 9 | 0 | 13 | 10 | 7 | 3 | 8 | 0 |
| 3 | 1 | 0 | ∞ | ∞ | 10 | 4 | 12 | 10 | 2 | 0 | 0 |
| 4 | 11 | 7 | 0 | ∞ | 7 | 4 | 3 | 12 | 4 | 4 | 0 |
| 5 | 1 | 0 | 12 | 9 | ∞ | 4 | 5 | 8 | 10 | 8 | 0 |
| 6 | 1 | 13 | 6 | 6 | 4 | ∞ | 7 | 0 | 7 | 3 | 0 |
| 7 | 0 | 10 | 14 | 5 | 5 | 7 | ∞ | 0 | 9 | 5 | 0 |
| 8 | 0 | 7 | 12 | 14 | 8 | 0 | 0 | ∞ | 8 | 10 | 0 |
| 9 | 11 | 1 | 2 | 4 | 8 | 5 | 7 | 6 | ∞ | 0 | 0 |
| 10 | 9 | 6 | 0 | 4 | 6 | 1 | 3 | 8 | 0 | ∞ | 0 |
| min | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |  |

Приводим матрицу

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| 1 | ∞ | 12 | 3 | 9 | 1 | 1 | 0 | 0 | 13 | 11 | 0 |
| 2 | 12 | ∞ | 2 | 5 | 0 | 13 | 10 | 7 | 3 | 8 | 2 |
| 3 | 1 | 0 | ∞ | ∞ | 10 | 4 | 12 | 10 | 2 | 0 | 0 |
| 4 | 11 | 7 | 0 | ∞ | 7 | 4 | 3 | 12 | 4 | 4 | 3 |
| 5 | 1 | 0 | 12 | 5 | ∞ | 4 | 5 | 8 | 10 | 8 | 1 |
| 6 | 1 | 13 | 6 | 2 | 4 | ∞ | 7 | 0 | 7 | 3 | 1 |
| 7 | 0 | 10 | 14 | 1 | 5 | 7 | ∞ | 0 | 9 | 5 | 0 |
| 8 | 0 | 7 | 12 | 10 | 8 | 0 | 0 | ∞ | 8 | 10 | 0 |
| 9 | 11 | 1 | 2 | 0 | 8 | 5 | 7 | 6 | ∞ | 0 | 0 |
| 10 | 9 | 6 | 0 | 0 | 6 | 1 | 3 | 8 | 0 | ∞ | 0 |
|  | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 2 | 0 |  |

Qmax = 3

Ветвление (2,5)

Оценка: 24+3 = 27

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 6 | 7 | 8 | 9 | 10 | min |  | 1 | 2 | 3 | 4 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 12 | 3 | 9 | 1 | 0 | 0 | 13 | 11 | 0 | 1 | ∞ | 12 | 3 | 9 | 1 | 0 | 0 | 13 | 11 |
| 3 | 1 | 0 | ∞ | ∞ | 4 | 12 | 10 | 2 | 0 | 0 | 3 | 1 | 0 | ∞ | ∞ | 4 | 12 | 10 | 2 | 0 |
| 4 | 11 | 7 | 0 | ∞ | 4 | 3 | 12 | 4 | 4 | 0 | 4 | 11 | 7 | 0 | ∞ | 4 | 3 | 12 | 4 | 4 |
| 5 | 1 | *∞* | 12 | 5 | 4 | 5 | 8 | 10 | 8 | 1 | 5 | 0 | *∞* | 11 | 4 | 3 | 4 | 7 | 9 | 7 |
| 6 | 1 | 13 | 6 | 2 | ∞ | 7 | 0 | 7 | 3 | 0 | 6 | 1 | 13 | 6 | 2 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 0 | 10 | 14 | 1 | 7 | ∞ | 0 | 9 | 5 | 0 | 7 | 0 | 10 | 14 | 1 | 7 | ∞ | 0 | 9 | 5 |
| 8 | 0 | 7 | 12 | 10 | 0 | 0 | ∞ | 8 | 10 | 0 | 8 | 0 | 7 | 12 | 10 | 0 | 0 | ∞ | 8 | 10 |
| 9 | 11 | 1 | 2 | 0 | 5 | 7 | 6 | ∞ | 0 | 0 | 9 | 11 | 1 | 2 | 0 | 5 | 7 | 6 | ∞ | 0 |
| 10 | 9 | 6 | 0 | 0 | 1 | 3 | 8 | 0 | ∞ | 0 | 10 | 9 | 6 | 0 | 0 | 1 | 3 | 8 | 0 | ∞ |
|  |  |  |  |  |  |  |  |  |  |  | min | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

h = 1

24 + 1 <= 27

Новая оценка 25

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 6 | 7 | 8 | 9 | 10 |  |
| 1 | ∞ | 12 | 3 | 9 | 1 | 0 | 0 | 13 | 11 | 0 |
| 3 | 1 | 0 | ∞ | ∞ | 4 | 12 | 10 | 2 | 0 | 0 |
| 4 | 11 | 7 | 0 | ∞ | 4 | 3 | 12 | 4 | 4 | 3 |
| 5 | 0 | *∞* | 11 | 4 | 3 | 4 | 7 | 9 | 7 | 3 |
| 6 | 1 | 13 | 6 | 2 | ∞ | 7 | 0 | 7 | 3 | 1 |
| 7 | 0 | 10 | 14 | 1 | 7 | ∞ | 0 | 9 | 5 | 0 |
| 8 | 0 | 7 | 12 | 10 | 0 | 0 | ∞ | 8 | 10 | 0 |
| 9 | 11 | 1 | 2 | 0 | 5 | 7 | 6 | ∞ | 0 | 0 |
| 10 | 9 | 6 | 0 | 0 | 1 | 3 | 8 | 0 | ∞ | 0 |
|  | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 2 | 0 |  |

Qmax =Q(4,3) = 3

Ветвление (4,3)

Оценка: 28

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 4 | 6 | 7 | 8 | 9 | 10 | min |  | 1 | 2 | 4 | 6 | 7 | 8 | 9 | 10 |
| 1 | ∞ | 12 | 9 | 1 | 0 | 0 | 13 | 11 | 0 | 1 | ∞ | 12 | 9 | 1 | 0 | 0 | 13 | 11 |
| 3 | 1 | 0 | ∞ | 4 | 12 | 10 | 2 | 0 | 0 | 3 | 1 | 0 | ∞ | 4 | 12 | 10 | 2 | 0 |
| 5 | 0 | *∞* | 4 | 3 | 4 | 7 | 9 | 7 | 0 | 5 | 0 | *∞* | 4 | 3 | 4 | 7 | 9 | 7 |
| 6 | 1 | 13 | 2 | ∞ | 7 | 0 | 7 | 3 | 0 | 6 | 1 | 13 | 2 | ∞ | 7 | 0 | 7 | 3 |
| 7 | 0 | 10 | 1 | 7 | ∞ | 0 | 9 | 5 | 0 | 7 | 0 | 10 | 1 | 7 | ∞ | 0 | 9 | 5 |
| 8 | 0 | 7 | 10 | 0 | 0 | ∞ | 8 | 10 | 0 | 8 | 0 | 7 | 10 | 0 | 0 | ∞ | 8 | 10 |
| 9 | 11 | 1 | 0 | 5 | 7 | 6 | ∞ | 0 | 0 | 9 | 11 | 1 | 0 | 5 | 7 | 6 | ∞ | 0 |
| 10 | 9 | 6 | 0 | 1 | 3 | 8 | 0 | ∞ | 0 | 10 | 9 | 6 | 0 | 1 | 3 | 8 | 0 | ∞ |
|  |  |  |  |  |  |  |  |  |  | min | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

h =0

25 <= 28

25 Новая оценка

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 4 | 6 | 7 | 8 | 9 | 10 |  |
| 1 | ∞ | 12 | 9 | 1 | 0 | 0 | 13 | 11 | 0 |
| 3 | 1 | 0 | ∞ | 4 | 12 | 10 | 2 | 0 | 0 |
| 5 | 0 | *∞* | 4 | 3 | 4 | 7 | 9 | 7 | 3 |
| 6 | 1 | 13 | 2 | ∞ | 7 | 0 | 7 | 3 | 1 |
| 7 | 0 | 10 | 1 | 7 | ∞ | 0 | 9 | 5 | 0 |
| 8 | 0 | 7 | 10 | 0 | 0 | ∞ | 8 | 10 | 0 |
| 9 | 11 | 1 | 0 | 5 | 7 | 6 | ∞ | 0 | 0 |
| 10 | 9 | 6 | 0 | 1 | 3 | 8 | 0 | ∞ | 0 |
|  | 0 | 1 | 0 | 1 | 0 | 0 | 2 | 0 |  |

Qmax = Q(5,1) = 3

Ветвление (5,1)

Оценка 28

Воизбежание внутреннего цикла запретить (1,2)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *2* | *4* | *6* | *7* | *8* | *9* | *10* | min |  | *2* | *4* | *6* | *7* | *8* | *9* | *10* |
| *1* | *∞* | *9* | *1* | *0* | *0* | *13* | *11* | 0 | *1* | *∞* | *9* | *1* | *0* | *0* | *13* | *11* |
| *3* | *0* | *∞* | *4* | *12* | *10* | *2* | *0* | 0 | *3* | *0* | *∞* | *4* | *12* | *10* | *2* | *0* |
| *6* | *13* | *2* | *∞* | *7* | *0* | *7* | *3* | 0 | *6* | *13* | *2* | *∞* | *7* | *0* | *7* | *3* |
| *7* | *10* | *1* | *7* | *∞* | *0* | *9* | *5* | 0 | *7* | *10* | *1* | *7* | *∞* | *0* | *9* | *5* |
| *8* | *7* | *10* | *0* | *0* | *∞* | *8* | *10* | 0 | *8* | *7* | *10* | *0* | *0* | *∞* | *8* | *10* |
| *9* | *1* | *0* | *5* | *7* | *6* | *∞* | *0* | 0 | *9* | *1* | *0* | *5* | *7* | *6* | *∞* | *0* |
| *10* | *6* | *0* | *1* | *3* | *8* | *0* | *∞* | 0 | *10* | *6* | *0* | *1* | *3* | *8* | *0* | *∞* |
|  |  |  |  |  |  |  |  |  | min | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

25 + 0 <= 28

Новая оценка 25

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *2* | *4* | *6* | *7* | *8* | *9* | *10* |  |
| *1* | *∞* | *9* | *1* | *0* | *0* | *13* | *11* | *0* |
| *3* | *0* | *∞* | *4* | *12* | *10* | *2* | *0* | *0* |
| *6* | *13* | *2* | *∞* | *7* | *0* | *7* | *3* | *2* |
| *7* | *10* | *1* | *7* | *∞* | *0* | *9* | *5* | *1* |
| *8* | *7* | *10* | *0* | *0* | *∞* | *8* | *10* | *0* |
| *9* | *1* | *0* | *5* | *7* | *6* | *∞* | *0* | *0* |
| *10* | *6* | *0* | *1* | *3* | *8* | *0* | *∞* | *0* |
|  | *1* | *0* | *1* | *0* | *0* | *2* | *0* |  |

Qmax = Q(6,8)=2

Ветвление (6,8)

Оценка 27

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *2* | *4* | *6* | *7* | *9* | *10* | MIN |  | *2* | *4* | *6* | *7* | *9* | *10* |
| *1* | *∞* | *9* | *1* | *0* | *13* | *11* | 0 | *1* | *∞* | *9* | *1* | *0* | *13* | *11* |
| *3* | *0* | *∞* | *4* | *12* | *2* | *0* | 0 | *3* | *0* | *∞* | *4* | *12* | *2* | *0* |
| *7* | *10* | *1* | *7* | *∞* | *9* | *5* | 1 | *7* | *9* | *0* | *6* | *∞* | *8* | *4* |
| *8* | *7* | *10* | ∞ | *0* | *8* | *10* | 0 | *8* | *7* | *10* | ∞ | *0* | *8* | *10* |
| *9* | *1* | *0* | *5* | *7* | *∞* | *0* | 0 | *9* | *1* | *0* | *5* | *7* | *∞* | *0* |
| *10* | *6* | *0* | *1* | *3* | *0* | *∞* | 0 | *10* | *6* | *0* | *1* | *3* | *0* | *∞* |
|  |  |  |  |  |  |  |  | MIN | 0 | 0 | 1 | 0 | 0 | 0 |

h = 2

27 <= 27

27 новая оценка

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *2* | *4* | *6* | *7* | *9* | *10* |
| *1* | *∞* | *9* | *0* | *0* | *13* | *11* |
| *3* | *0* | *∞* | *3* | *12* | *2* | *0* |
| *7* | *9* | *0* | *5* | *∞* | *8* | *4* |
| *8* | *7* | *10* | ∞ | *0* | *8* | *10* |
| *9* | *1* | *0* | *4* | *7* | *∞* | *0* |
| *10* | *6* | *0* | *0* | *3* | *0* | *∞* |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | *2* | *4* | *6* | *7* | *9* | *10* |  |
| *1* | *∞* | *9* | *0* | *0* | *13* | *11* | *0* |
| *3* | *0* | *∞* | *3* | *12* | *2* | *0* | *0* |
| *7* | *9* | *0* | *5* | *∞* | *8* | *4* | *4* |
| *8* | *7* | *10* | ∞ | *0* | *8* | *10* | *7* |
| *9* | *1* | *0* | *4* | *7* | *∞* | *0* | *0* |
| *10* | *6* | *0* | *0* | *3* | *0* | *∞* | *0* |
|  | *1* | 0 | *0* | *0* | *2* | *0* |  |

Qmax = Q(8,7) = 7

Ветвление(8,7)

Оценка 27+7 = 34

Запретить(7,6)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *2* | *4* | *6* | *9* | *10* | MIN |  | *2* | *4* | *6* | *9* | *10* |
| *1* | *∞* | *9* | *0* | *13* | *11* | 0 | *1* | *∞* | *9* | *0* | *13* | *11* |
| *3* | *0* | *∞* | *3* | *2* | *0* | 0 | *3* | *0* | *∞* | *3* | *2* | *0* |
| *7* | *9* | *0* | ∞ | *8* | *4* | 0 | *7* | *9* | *0* | ∞ | *8* | *4* |
| *9* | *1* | *0* | *4* | *∞* | *0* | 0 | *9* | *1* | *0* | *4* | *∞* | *0* |
| *10* | *6* | *0* | *0* | *0* | *∞* | 0 | *10* | *6* | *0* | *0* | *0* | *∞* |
|  |  |  |  |  |  |  | MIN | 0 | 0 | 0 | 0 | 0 |

27 + 0 <= 34

Новая оценка 27

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 2 | 4 | 6 | 9 | 10 |  |
| 1 | ∞ | 9 | 0 | 13 | 11 | 9 |
| 3 | 0 | ∞ | 3 | 2 | 0 | 0 |
| 7 | 9 | 0 | ∞ | 8 | 4 | 4 |
| 9 | 1 | 0 | 4 | ∞ | 0 | 0 |
| 10 | 6 | 0 | 0 | 0 | ∞ | 0 |
|  | 1 | 0 | 0 | 2 | 0 |  |

Qmax = 9

Ветвление (1,6)

Оценка 36

Запретить (7,2)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 2 | 4 | 9 | 10 | MIN |  | *2* | *4* | *9* | *10* |
| 3 | 0 | ∞ | 2 | 0 | 0 | *3* | *0* | *∞* | *2* | *0* |
| 7 | *∞* | 0 | 8 | 4 | 0 | *7* | *∞* | *0* | *8* | *4* |
| 9 | 1 | 0 | ∞ | 0 | 0 | *9* | *1* | *0* | *∞* | *0* |
| 10 | 6 | 0 | 0 | ∞ | 0 | *10* | *6* | *0* | *0* | *∞* |
|  |  |  |  |  |  | MIN | 0 | 0 | 0 | 0 |

27 <= 36

Новая оценка 27

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *2* | *4* | *9* | *10* |  |
| *3* | *0* | *∞* | *2* | *0* | 0 |
| *7* | *∞* | *0* | *8* | *4* | 4 |
| *9* | *1* | *0* | *∞* | *0* | 0 |
| *10* | *6* | *0* | *0* | *∞* | 0 |
|  | 1 | 0 | *2* | 0 |  |

Qmax = 4

Ветвление(7,4)

Оценка 27+4=31

Запретить(3,2)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *2* | *4* | *9* | *10* | MIN |  | *2* | *4* | *9* | *10* |
| *3* | ∞ | *∞* | *2* | *0* | 0 | *3* | ∞ | *∞* | *2* | *0* |
| *7* | *∞* | *0* | *8* | *4* | 0 | *7* | *∞* | *0* | *8* | *4* |
| *9* | *1* | *0* | *∞* | *0* | 0 | *9* | *1* | *0* | *∞* | *0* |
| *10* | *6* | *0* | *0* | *∞* | 0 | *10* | *6* | *0* | *0* | *∞* |
|  |  |  |  |  |  | MIN | 1 | 0 | 0 | 0 |

h = 1

27+1 = 28 <= 31

Новая оценка 28

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *2* | *4* | *9* | *10* |
| *3* | ∞ | *∞* | *2* | *0* |
| *7* | *∞* | *0* | *8* | *4* |
| *9* | *0* | *0* | *∞* | *0* |
| *10* | *5* | *0* | *0* | *∞* |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *2* | *4* | *9* | *10* |  |
| *3* | ∞ | *∞* | *2* | *0* | 2 |
| *7* | *∞* | *0* | *8* | *4* | 4 |
| *9* | *0* | *0* | *∞* | *0* | 0 |
| *10* | *5* | *0* | *0* | *∞* | 0 |
|  | 5 | *0* | *2* | 0 |  |

Qmax = Q(9,2) =5

Ветвление(9,2)

Оценка 33

Запретить(3,9)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | *4* | *9* | *10* | MIN |  | *4* | *9* | *10* |
| *3* | *∞* | ∞ | *0* | 0 | *3* | *∞* | ∞ | *0* |
| *7* | *0* | *8* | *4* | 0 | *7* | *0* | *8* | *4* |
| *10* | *0* | *0* | *∞* | 0 | *10* | *0* | *0* | *∞* |
|  |  |  |  |  | MIN | 0 | 0 | 0 |

H =0

28 <= 33

28 новая оценка

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *4* | *9* | *10* |  |
| *3* | *∞* | ∞ | *0* | ***0*** |
| *7* | *0* | *8* | *4* | 4 |
| *10* | *0* | *0* | *∞* | 0 |
|  | 0 | 8 | 4 |  |

Qmax = Q(10,9) = 8

Оценка 36

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *4* | *10* | MIN |  | *4* | *10* |
| *3* | *∞* | *0* | 0 | *3* | *∞* | *0* |
| *7* | *0* | *4* | 0 | *7* | *0* | *4* |
|  |  |  |  | MIN | 0 | 0 |

28 + 0 <=36

28 новая оценка

|  |  |  |
| --- | --- | --- |
|  | *4* | *10* |
| *3* | *∞* | *0* |
| *7* | *0* | *4* |

Маршрут: 9->2 ->5->1->6->8->7->4->3->10->9

S = 28

Оптимально

# Бла бла